LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

SECOND SEMESTER – **APRIL 2023**

PST 2501 – ESTIMATION THEORY

Date: 09-05-2023 Dept. No. Max. : 100 Marks
Time: 01:00 PM - 04:00 PM

SECTION - A

Answer ALL questions. Each carries TWO marks. **(10 x 2 = 20 marks)**

1. Explain the problem of Point Estimation.
2. State the different approaches to find UMVUE.
3. If δ is an UMVUE, then show that δ +2 is also an UMVUE.
4. Let X ~ B (1, θ), θ = 0.2, 0.3. Find MLE of θ.
5. Let X_1, X_2 be iid N (θ, 1), θ ∈ R. Is $X_1 – X_2$ an ancillary statistic?
6. Let X_1, X_2 be iid B (1, θ), θ ∈ (0, 1). Examine whether the points (0, 0) & (0, 1) are likelihood equivalent.
7. Define Minimum Variance Bound Estimator.
8. Prove that the family {N (0, σ^2), $\sigma^2 > 0$ } is not complete.
9. Define Consistency of a sequence of estimators.
10. What is meant by Conjugate Prior?

SECTION – B

Answer any FIVE questions. Each carries EIGHT marks. **(5 x 8 = 40 marks)**

11. Let X_1, X_2 be iid P (λ) RVs. Verify if (i) $T_1 = X_1 + X_2$ and (ii) $T_2 = X_1 + 2X_2$ are sufficient for ‘λ’.
12. Let X ~ B (1, θ), θ = 0.1, 0.2, 0.3 and let g(θ) = θ^2 , θ = 0.1, 0.2, 0.3. Verify if the class of unbiased estimators of ‘g’ is empty.
13. Let X be discrete with PMF
    $$P_\theta (X = x) = \begin{cases} \theta, & x = -1, \\ (1-\theta)^2\, \theta^x, & x = 0, 1, 2, \ldots; 0 < \theta < 1. \end{cases}$$
    Find the class of UMVUEs.
14. Write the statement of Basu’s theorem and establish it.
15. Verify if bounded completeness imply completeness of a family of distributions of X.
16. Let $X_1, X_2, \ldots, X_n$ be a sample from N(μ, σ^2), where both μ and σ^2 *are* unknown. Find two jointly sufficient statistics for the parameter (μ, σ^2).
17. Let $X_1, X_2, \ldots, X_n$ be iid B (1, p) random variables. Then prove that T = $\sum_1^n X_i$ is a complete sufficient statistic.
18. Let X_1, X_2 be a random sample from N (θ, 1), θ ϵ R. Find Cramer-Rao Lower Bound for estimating θ. Hence check if $\delta^*(\underline{X}) = (X_1 + X_2) / 2$ is UMVUE of θ.

SECTION – C

Answer any TWO questions. Each carries TWENTY marks. **(2 x 20 = 40 marks)**

19. (a) Let u_g be the class of unbiased estimators of ‘g’ with finite second moment. Let u_0 be the class of unbiased estimators of ‘0’ with finite second moment. Prove that $\delta^* \epsilon\, u_g$ is a UMVUE of ‘g’ if and only if δ^* is uncorrelated with every u ϵ u_0. (10)
    (b) State and prove a criterion for determining a sufficient statistics. (10)
20. (a) Let X_1, X_2 be a random sample from N(θ, 1) , θ ϵ R. Show that $\alpha_1 X_1 + \alpha_2 X_2$ is sufficient if and only if $\alpha_1 = \alpha_2$, where α_1, α_2 ϵ R. (10)

(b) Let $X_1, X_2, \ldots, X_n$ be a random sample from $E(\mu, \sigma)$, $\mu \in R$, $\sigma > 0$. Find MLE of (μ, σ). (10)

21. (a) Let $X \sim DU\{\theta, \theta + 1\}$, $\theta = 1, 2$. Prove that no non-constant function possesses UMVUE. (10)

(b) State and prove Invariance Property of CAN estimator. (10)

22. (a) Let $X \sim N(\mu, 1)$, $\mu \in R$ and let the prior distribution of μ be $N(0, 1)$. Find the Bayes'estimator of μ with respect to squared error loss. (10)

(b) Let $X_1, X_2, \ldots, X_n$ be a random sample from $U(0, \theta)$, $\theta > 0$. Find a minimal sufficient statistic. (10)

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